Impredicative Concurrent Abstract Predicates

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Introduction

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- A logic for modular reasoning about partial correctness of concurrent, higher-order, reentrant, imperative code.

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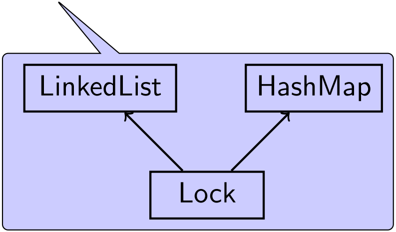

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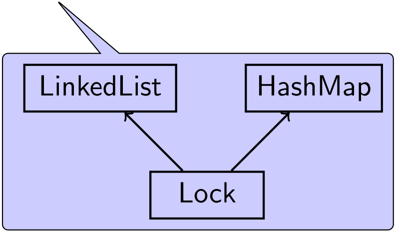

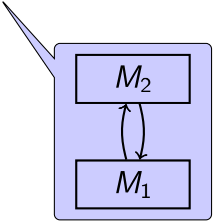

A Modular Lock Specification

$$\exists \mathsf{isLock}, \mathsf{locked} : \mathsf{Val} \times \mathsf{Prop} \to \mathsf{Prop}.\ \forall \mathsf{R} : \mathsf{Prop}.\ \mathsf{stable}(\mathsf{R}) \Rightarrow$$

$$\begin{array}{rcl} \{\mathsf{R}\} & \texttt{new Lock()} & \{\mathsf{isLock}(\mathsf{ret}, \mathsf{R})\} \\ \{\mathsf{isLock}(\mathsf{x}, \mathsf{R})\} & \texttt{x.Acquire()} & \{\mathsf{locked}(\mathsf{x}, \mathsf{R}) * \mathsf{R}\} \\ \{\mathsf{locked}(\mathsf{x}, \mathsf{R}) * \mathsf{R}\} & \texttt{x.Release()} & \{\mathsf{isLock}(\mathsf{x}, \mathsf{R})\} \end{array}$$

$$\forall \mathsf{x} : \mathsf{Val}.\ \mathsf{isLock}(\mathsf{x}, \mathsf{R}) \Leftrightarrow \mathsf{isLock}(\mathsf{x}, \mathsf{R}) * \mathsf{isLock}(\mathsf{x}, \mathsf{R})$$

$$\forall \mathsf{x} : \mathsf{Val}.\ \mathsf{stable}(\mathsf{isLock}(\mathsf{x}, \mathsf{R})) \wedge \mathsf{stable}(\mathsf{locked}(\mathsf{x}, \mathsf{R}))$$

A Modular Lock Specification

Standard sep. logic lock specification
The resource invariant R describes the resources protected by the lock.

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A Modular Lock Specification

Third-order quantification.

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A Modular Lock Specification

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A Modular Lock Specification

Second-order quantification.

$$\forall \mathsf{R} : \mathsf{Prop}.\ \exists \mathsf{isLock}, \mathsf{locked} : \mathsf{Val} \to \mathsf{Prop}.\ \mathsf{stable}(\mathsf{R}) \Rightarrow$$

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A Modular Lock Specification

This specification might suffice for layering of abstractions, but **not** for all **recursive** abstractions.

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Recursive Abstractions

Reentrant Event Loop Library

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delegate void handler();

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Event handlers are allowed to emit events!

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A library that allows us to close Landin's Knot / perform recursion through the store.

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Realistic examples of this form:
libevent, Node.js, Twisted, ...
C5, GUI libraries, Joins library, ...

Recursive Abstractions

Event Loop Memory Safety Specification

$\exists \mathsf{eloop} : \mathsf{Val} \rightarrow \mathsf{Prop}.$

$$\begin{array}{rcl}
\{\mathsf{emp}\} & \texttt{new EventLoop()} & \{\mathsf{eloop}(\mathsf{ret})\} \\
\{\mathsf{eloop}(\mathsf{x})\} & \texttt{x.loop()} & \{\mathsf{eloop}(\mathsf{x})\} \\
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\{\mathsf{eloop}(\mathsf{x}) * P\} & \texttt{x.when(f)} & \{\mathsf{eloop}(\mathsf{x})\}
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$$\forall \mathsf{x} : \mathsf{Val}.\ \mathsf{eloop}(\mathsf{x}) \Leftrightarrow \mathsf{eloop}(\mathsf{x}) * \mathsf{eloop}(\mathsf{x})$$

where $P = f \mapsto \{\mathsf{emp}\}\{\mathsf{emp}\}$

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Event handler must run without any resources and emitting an event requires an eloop(x) resource!

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- Since handlers can signal events, the footprint of handlers include the footprint of their event loop.
- The footprint of an event loop is thus recursively defined.

Recursive Abstractions

Verifying a lock-based event loop implementation

- Imagine an implementation that maintains a set of signal handlers and a set of pending signals, protected by a lock:

```
class EventLoop : IEventLoop {
  private Lock lock;
  private Set<handler> handlers;
  private Set<signal> signals;

  ...
}
```

- Tying Landin's Knot using a reference protected by a lock.

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- We define eloop using **guarded recursion** and the **third-order** isLock representation predicate:

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Must be non-expansive!

iCAP

Reasoning about shared state

- Following CAP, iCAP extends separation logic with shared regions and protocols to govern shared state.

- The state is split into a local part and shared regions.

local	region 1	region 2

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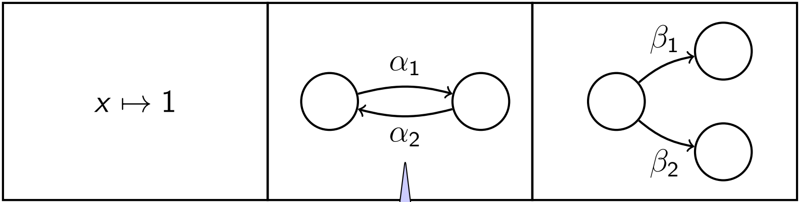

A labelled transition system specifies the possible **abstract states** of the shared region.

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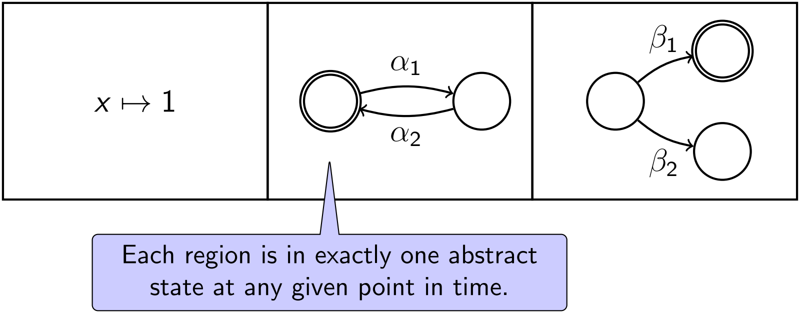

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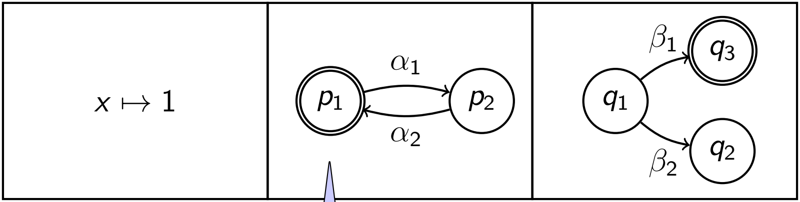

A predicate that defines the resources owned by the shared region in the given abstract state.

A Modular Lock Specification

Verifying a spinlock implementation

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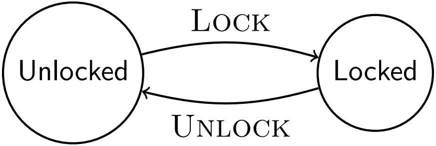

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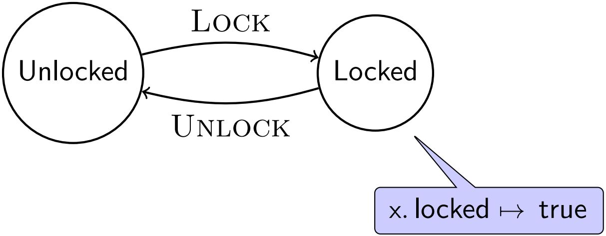

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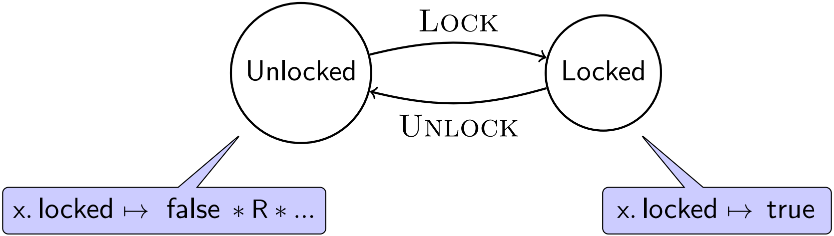
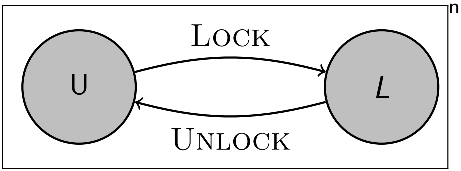


A Modular Lock Specification

Verifying a spinlock implementation

Formally

$$\mathsf{isLock}(x, R) = \exists n : \mathsf{RId}.\ [\textsc{Lock}]^{n}_{-} * \mathsf{rintr}(I(x, R, n), n)$$

$*$ 

where

$$I(x, R, n)(s) = \begin{cases} x.\mathsf{locked} \mapsto \mathsf{true} & \text{if } s = \mathsf{L} \\ x.\mathsf{locked} \mapsto \mathsf{false} * R * [\textsc{Unlock}]^{n}_{1} & \text{if } s = \mathsf{U} \end{cases}$$

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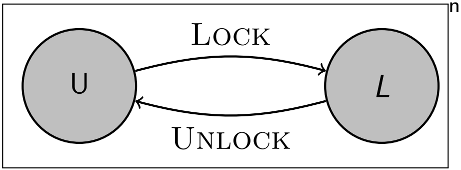
Verifying a spinlock implementation

Formally

$\mathsf{rintr} : (\mathsf{SId} \to \mathsf{Prop}) \times \mathsf{RId} \to \mathsf{Prop}$

$$\mathsf{isLock}(\mathsf{x}, \mathsf{R}) = \exists \mathsf{n} : \mathsf{RId}.\ [\textsc{Lock}]^{\mathsf{n}}_{\_} * \mathsf{rintr}(\mathsf{I}(\mathsf{x}, \mathsf{R}, \mathsf{n}), \mathsf{n})$$

$*$ 

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Model

Impredicative protocols introduce a circularity

- rintr(I, n) : Prop, asserts that the interpretation of the abstract states of region n are given by I : SId $\to$ Prop

$$\mathit{Prop} \cong \mathcal{P}^{\uparrow}(\ldots \times (\mathit{RId} \times \mathit{SId} \rightharpoonup_{\mathit{fin}} \mathit{Prop}))$$

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- iCAP function space is the topos of trees function space:

$$[\![\tau \rightarrow \sigma]\!] = [\![\tau]\!] \rightarrow [\![\sigma]\!]$$

iCAP function space

Topos of trees function space

Related work

CAP

- [Dodds et al., POPL 2011] was unsound, because the authors broke the circularity introduced by impredicative protocols, but reasoned as if they had solved it.
- In HOCAP [ESOP 2013] we broke the circularity and introduced a predicative stratification to ensure soundness.

CaReSL [Turon et al., ICFP 2013]

- Model related to iCAP model, but logic is only second-order, so types of CaReSL can be interpreted as constant sets (in iCAP they are variable sets, objects in topos of trees).

Conclusion

Recursive abstractions

- Recursive abstractions are **useful** and ubiquitous in higher-order code with effects!
- We can reason about recursive abstractions using higher-order specifications and guarded recursion.

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- A logic for modular reasoning about partial correctness of concurrent, higher-order, reentrant, imperative code.